

主題 13 有理化分母

◎基礎精選◎

1. 試比較下列各數的大小關係：

(1) $a = \sqrt{6} + 2\sqrt{2}$, $b = 2\sqrt{7}$, $c = 3 + \sqrt{5}$,

大小關係為_____。

(2) $a = \sqrt{17} - \sqrt{13}$, $b = 4 - 2\sqrt{3}$, $c = \sqrt{15} - \sqrt{11}$,

大小關係為_____。



[數位延伸]

2. 將 $K = \frac{2}{\sqrt[3]{4} + \sqrt[3]{2} + 1}$ 的分母有理化後，得 K 值為_____。



1.【答】：(1) $b > a > c$ (2) $c > b > a$

【解】：(1) $a^2 = 14 + 2\sqrt{48}, b^2 = 28 = 14 + 2\sqrt{49}, c^2 = 14 + 2\sqrt{45}$

$$\therefore b^2 > a^2 > c^2 \Rightarrow b > a > c$$

(2) $a \times (\sqrt{17} + \sqrt{13}) = 4, b \times (\sqrt{16} + \sqrt{12}) = 4, c \times (\sqrt{15} + \sqrt{11}) = 4$

$$\therefore \sqrt{17} + \sqrt{13} > \sqrt{16} + \sqrt{12} > \sqrt{15} + \sqrt{11} \therefore c > b > a$$

2.【答】： $2\sqrt[3]{2} - 2$

【解】： $K = \frac{2}{\sqrt[3]{4} + \sqrt[3]{2} + 1} \times \frac{\sqrt[3]{2} - 1}{\sqrt[3]{2} - 1} = \frac{2(\sqrt[3]{2} - 1)}{2 - 1} = 2\sqrt[3]{2} - 2$

3. 試求下列各數的倒數

(1) $\sqrt{2}-1$

(2) $\sqrt{2}+\sqrt{3}$

[數位延伸]

(3) $\sqrt[3]{4}-1$

UZGBN



4. 求值 $\frac{1}{\sqrt{1}+\sqrt{3}} + \frac{1}{\sqrt{3}+\sqrt{5}} + \frac{1}{\sqrt{5}+\sqrt{7}} + \cdots + \frac{1}{\sqrt{47}+\sqrt{49}}$

WCYPZ



3.【答】 : (1) $\sqrt{2}+1$ (2) $\sqrt{3}-\sqrt{2}$ (3) $\frac{2\sqrt[3]{2}+\sqrt[3]{4}+1}{3}$

【解】 : (1) $\frac{1}{\sqrt{2}-1} = \sqrt{2}+1$

(2) $\frac{1}{\sqrt{2}+\sqrt{3}} = \sqrt{3}-\sqrt{2}$

(3) $\frac{1}{\sqrt[3]{4}-1} = \frac{\sqrt[3]{16}+\sqrt[3]{4}+1}{(\sqrt[3]{4}-1)(\sqrt[3]{16}+\sqrt[3]{4}+1)} = \frac{2\sqrt[3]{2}+\sqrt[3]{4}+1}{3}$

4.【答】 : 3

【解】 :
$$\begin{aligned} & \frac{1}{\sqrt{1}+\sqrt{3}} + \frac{1}{\sqrt{3}+\sqrt{5}} + \frac{1}{\sqrt{5}+\sqrt{7}} + \cdots + \frac{1}{\sqrt{47}+\sqrt{49}} \\ &= \frac{\sqrt{3}-1}{2} + \frac{\sqrt{5}-\sqrt{3}}{2} + \frac{\sqrt{7}-\sqrt{5}}{2} + \cdots + \frac{\sqrt{49}-\sqrt{47}}{2} \\ &= \frac{-1+\sqrt{49}}{2} = 3 \end{aligned}$$

★能力進階★

1. 設 $F(x) = 1 + \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{3}} + \frac{1}{\sqrt{4}} + \cdots + \frac{1}{\sqrt{100}} = a + b$, a 為整數,

b 為小數, 求 $a = \underline{\hspace{2cm}}$ 。

提示: $\sqrt{n+1} - \sqrt{n} = \frac{1}{\sqrt{n+1} + \sqrt{n}} < \frac{1}{\sqrt{n} + \sqrt{n}} = \frac{1}{2\sqrt{n}}$

同理 $\frac{1}{2\sqrt{n}} < \sqrt{n} - \sqrt{n-1}$, 所以 $\sqrt{n+1} - \sqrt{n} < \frac{1}{2\sqrt{n}} < \sqrt{n} - \sqrt{n-1}$



[數位延伸]

2. 試求 $11x^3 - 3x^2 - 3x - 1 = 0$ 的實根。



3. 化簡 $\sqrt{2+\sqrt{3}} \cdot \sqrt{2+\sqrt{2+\sqrt{3}}} \cdot \sqrt{2+\sqrt{2+\sqrt{2+\sqrt{3}}}} \cdot \sqrt{2-\sqrt{2+\sqrt{2+\sqrt{3}}}}$ 。



1.【答】 :18

$$\begin{aligned} \text{【解】 : } & \sqrt{3}-\sqrt{2} < \frac{1}{2\sqrt{2}} < \sqrt{2}-\sqrt{1} \\ & \vdots \\ & +) \sqrt{101}-\sqrt{100} < \frac{1}{2\sqrt{100}} < \sqrt{100}-\sqrt{99} \\ & \hline & \sqrt{101}-\sqrt{2} < \frac{1}{2}(F(x)-1) < \sqrt{100}-1 \\ & \Rightarrow 2(\sqrt{101}-\sqrt{2})+1 \doteq 18.2 \\ & \Rightarrow F(x)=18. \blacksquare \Rightarrow a=18 \end{aligned}$$

2.【答】 : $\frac{2\sqrt[3]{18}+\sqrt[3]{12}+1}{11}$

$$\begin{aligned} \text{【解】 : } & 11x^3-3x^2-3x-1=0 \\ & \Rightarrow 12x^3=x^3+3x^2+3x+1 \\ & \Rightarrow 12x^3=(x+1)^3 \\ & \Rightarrow \sqrt[3]{12}x=x+1 \quad (x \text{ 為實根, 故可直接開三次方根}) \\ & \Rightarrow x=\frac{1}{\sqrt[3]{12}-1}=\frac{\sqrt[3]{144}+\sqrt[3]{12}+1}{11}=\frac{2\sqrt[3]{18}+\sqrt[3]{12}+1}{11} \end{aligned}$$

3.【答】 :1

$$\begin{aligned} \text{【解】 : } & \sqrt{2+\sqrt{3}} \cdot \sqrt{2+\sqrt{2+\sqrt{3}}} \cdot \sqrt{2+\sqrt{2+\sqrt{2+\sqrt{3}}}} \cdot \sqrt{2-\sqrt{2+\sqrt{2+\sqrt{3}}}} \\ & = \sqrt{2+\sqrt{3}} \cdot \sqrt{2+\sqrt{2+\sqrt{3}}} \cdot \sqrt{4-(2+\sqrt{2+\sqrt{3}})} \\ & = \sqrt{2+\sqrt{3}} \cdot \sqrt{2+\sqrt{2+\sqrt{3}}} \cdot \sqrt{2-\sqrt{2+\sqrt{3}}} \\ & = \sqrt{2+\sqrt{3}} \cdot \sqrt{4-(2+\sqrt{3})} \\ & = \sqrt{2+\sqrt{3}} \cdot \sqrt{2-\sqrt{3}} \\ & = \sqrt{4-3} \\ & = 1 \end{aligned}$$