



主題 9 Σ 運算

ZSIC



【觀念一】 Σ 符號介紹

- (1) Σ 讀做 sigma
- (2) Σ 表連加的意義：讀做 summation
- (3) $a_1 + a_2 + a_3 + \cdots + a_{18} = S_{18} = \Sigma$
- (4) $b_1 + b_2 + b_3 + \cdots + b_{70} = S_{70} = \Sigma$
- (5) $a_1 + a_2 + \cdots + a_n = S_n = \Sigma$
- (6) $b_1 + b_2 + \cdots + b_n + \cdots = \Sigma$
- (7) $\sum_{k=1}^8 t =$
- (8) $\sum_{k=1}^5 (2k-1) =$
- (9) $\sum_{k=1}^{\infty} \left(\frac{1}{3}\right)^k =$ (僅需列出格式不用寫出答案)

【觀念二】 $\sum_{k=1}^n a_k =$

- | | |
|----------|----------|
| (1) 項數 = | (2) 下限 = |
| (3) 上限 = | (4) 變數是 |

《說例》將下列各級數用 Σ 表示：(不需求和)

$$(1) 1 \cdot 4 + 2 \cdot 5 + 3 \cdot 6 + 4 \cdot 7 + 5 \cdot 8 + \cdots + (\text{第 } n \text{ 項})$$
$$=$$

$$(2) \frac{1 \cdot 2}{2 \cdot 3} + \frac{2 \cdot 2^2}{3 \cdot 4} + \frac{3 \cdot 2^3}{4 \cdot 5} + \frac{4 \cdot 2^4}{5 \cdot 6} + \frac{5 \cdot 2^5}{6 \cdot 7} + \cdots + (\text{第 } n \text{ 項})$$
$$=$$

$$(3) 1 \cdot n + 2 \cdot (n-1) + 3 \cdot (n-2) + 4 \cdot (n-3) + \cdots + (n-1) \cdot 2 + n \cdot 1$$
$$=$$

【觀念三】 $\sum_{k=1}^n c \cdot a_k = c \cdot a_1 + c \cdot a_2 + \cdots + c \cdot a_n = c(a_1 + a_2 + \cdots + a_n)$

$$= c \sum_{k=1}^n a_k$$

【觀念四】 $\sum_{k=1}^n (a_k + b_k) = (a_1 + b_1) + (a_2 + b_2) + \cdots + (a_n + b_n)$

$$= (a_1 + a_2 + \cdots + a_n) + (b_1 + b_2 + \cdots + b_n)$$

$$= \sum_{k=1}^n a_k + \sum_{k=1}^n b_k$$

$$\begin{aligned}
 \text{【觀念五】 } \sum_{k=1}^n (a_k - b_k) &= (a_1 - b_1) + (a_2 - b_2) + \cdots + (a_n - b_n) \\
 &= (a_1 + a_2 + \cdots + a_n) - (b_1 + b_2 + \cdots + b_n) \\
 &= \sum_{k=1}^n a_k - \sum_{k=1}^n b_k
 \end{aligned}$$

$$\begin{aligned}
 \text{【觀念六】 } \sum_{k=1}^n (a_k \cdot b_k) &= (a_1 b_1) + (a_2 b_2) + \cdots + (a_n b_n) \\
 &\neq (a_1 + a_2 + \cdots + a_n)(b_1 + b_2 + \cdots + b_n) \\
 \therefore \sum_{k=1}^n (a_k \cdot b_k) &\neq \sum_{k=1}^n a_k \times \sum_{k=1}^n b_k
 \end{aligned}$$

$$\begin{aligned}
 \text{【觀念七】 } \sum_{k=1}^n \frac{a_k}{b_k} &= \frac{a_1}{b_1} + \frac{a_2}{b_2} + \cdots + \frac{a_n}{b_n} \neq \frac{a_1 + a_2 + \cdots + a_n}{b_1 + b_2 + \cdots + b_n} \\
 \therefore \sum_{k=1}^n \frac{a_k}{b_k} &\neq \frac{\sum_{k=1}^n a_k}{\sum_{k=1}^n b_k}
 \end{aligned}$$